Lesson 52: Scalar Curvature Part I

Introduction

Let's continue with our exploration of some of the tensors associated with the Riemann tensor, the Riemann curvature tensor. Last lesson we derived an equation that looked a little like this:

$$\ddot{V}_N = R_{lk} X^l X^k V_N \tag{1}$$

Our goal, understand what our goal was, our goal is an interpretation, the idea being that we understand that the Riemann tensor is something that exists and we've thought very closely about why $R^{\mu}{}_{\nu\alpha\beta}$, we studied this guy we talked about it in terms of separating geodesics that was a geodesic interpretation method and we also derived it in terms of Parallel transport of a vector around a little square remember we had a vector here and we parallel transported it this way and, this way, and we got a difference between the vector at that point so as we derived this structure, as we took the properties of space-time the properties of the manifold that all flow from the connection right, everything flows from the connection, the connection implied this property and it directly applied the existence of this $(1,3)$ tensor right, and so the derivation itself gave us an interpretation right, we're able to think of these things as this tensor the Riemann curvature tensor can be interpreted as the machine, the tensor machine that absorbs certain vectors and tells us about the acceleration between two nearby geodesics, likewise we could say if we Parallel transport a vector from one point around a little loop, we know that it doesn't end up at this at the opposing part of the loop with being the same vector or likewise, another way of thinking of it is taking a vector and parallel transporting it around a loop and when it returns it's not the same vector.

Then the idea I was just saying it would come around and to be back here and it's still a deviation either way, the point being that this tensor can be understood as informing us about properties of space-time that are actually pretty easy to understand once you understand geodesics, the Geodesic deviation and the Parallel transport idea, is pretty simple thing to capture and we have a good way therefore of interpreting what this tensor is all about and that interpretation dropped directly from its derivation. That was not true for the Ricci tensor and the entire last lecture was to go from the Riemann tensor and play around with ideas in space-time and ultimately derive this notion (1) that now does let us interpret the Ricci tensor at least in one way and we basically created the interpretation that if we establish, let's remove it over here, if we established a little volume in space-time a spherical volume in space time and we asserted that each of the dust particles in that spacetime was oriented or had a motion in space-time, a spacetime path at some instant, $t=0$ say, and that was given by a vector X then we know that whatever the volume, whatever the volume of that sphere was, that volume is going to accelerate and that acceleration, that accelerating volume is given by this number right here (1) and by this expression and so ergo the Riemann or the Ricci tensor where that first contraction or the one-and-only contraction of the Riemann curvature tensor, informs us something about how the volume, the size of the volume accelerates given the initial volume.

Intuition

Now the bigger word that's commonly thrown about in this process and that word is intuition and I use it sometimes, I shouldn't though, intuition is not the right word here, intuition is something that whenever most of the time when people use intuition, what they're trying to do is they're trying to escape the math they're trying to not dig in and study and understand the origins of these things in its minute detail and it's pure abstract form, they want to skip all that and then just get me an intuitive picture of what this is all about and they try to learn the math with some intuitive guide they think some intuitive guide exists that'll get you there and having studied this for many years and have actually been in that trap at one point I can assure you it's a fool's errand, there's no way the proper way to do this is to study the detailed mathematics is complex and as abstract as it is and then go back and make sure you have an interpretation of that, there's no there's no way through it and this word is typically more of this I do of you know maybe I can escape understanding the math this word says now that I know the math what is it trying to tell me and that's where you want to be when you study this material or any any scientific material at all.

Now that this picture is to start a path to do the same thing and come up with an interpretation of the Ricci scalar or the curvature scalar, I guess I should call it, and the curvature scalar is actually a metric derived tensor because the curvature scalar is defined by taking the Ricci tensor right and contracting it with the metric to get a contraction of the Ricci tensor which is just a scalar that we call $\boldsymbol{R}$:

$$g^{lk}\boldsymbol{R}_{lk}=\boldsymbol{R}^{l}{}_{l}=\boldsymbol{R} \tag{2}$$

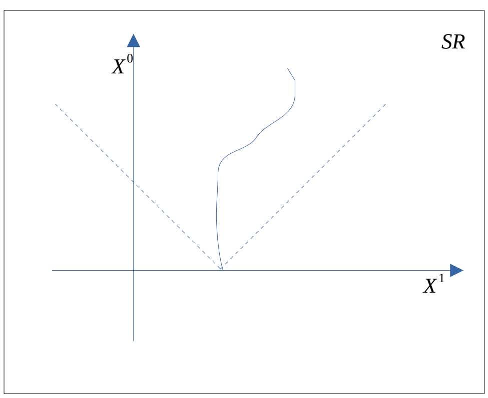
Notice that you can't do this without a metric, you need the metric to do this. That was not the case back here by the way right, it seems like it but we derived this thing and you can go back and do this, when we derive this object $R^{\mu}{}_{\nu\alpha\beta}$ it depended on a connection, if you give me a connection inside this manifold that will be sufficient to define this creature, the Riemann tensor and it's the notion of General relativity is such is that there is a metric and that metric defines the unique connection called the Levi-Civita connection or the metric connection is another way of saying it so we in General relativity we are presuming the existence of a metric but that's an additional feature right, that's an additional a manifold will have can have a connection that's unrelated to any metric, you can have a metric free manifold with a connection to find on it and you'll still have a Riemann tensor and you'll still have a Ricci tensor because you can always contract on this Riemann tensor but having a Ricci tensor to get to get the scalar you need to raise an index which is equivalent to saying you need to have unambiguous access to the co-vector space right, you have to convert one of these vector slots into a co-vector slot and to do that, the only way to do that canonically, is with a metric as far as I know so let's just say I'm claiming that you need this metric to make this last step it so it's a metric induced tensor.

Now the good news is that this great paper and this paper that I'm really coming to enjoy or at least I'm enjoying the parts I'm reading it's quite long so I'm not claiming to read the whole thing but the parts I'm reading that I enjoy a lot does have a very serious discussion of the interpretation of the curvature scalar just like it did or the Ricci tensor so we're going to go through that but to get there we have to go through another concept, we have to go through another notion that is very important in the proof and that notion is the notion of normal coordinates often called Riemann Normal coordinates.

Riemann normal coordinates

This is one of those topics I that is presented it's actually pretty this is an important topic it's the problem with this topic is that it doesn't necessarily need to be known as a precursor to studying the basic things of elementary General relativity, you don't need it to understand black holes you don't needed to understand geodesics, you don't need it to understand anything specific, on the other hand it's important I think for a real good grasp of the Equivalence principle right, I think understanding this as Equivalence principle is good it also gives a good feel for how to imagine laboratory frames in a curved space-time so it is a really pretty important idea, it's got a related concept called Fermi normal coordinates and I think what I'll do in this lecture is I'll discuss both of these just as an introduction. When Riemann normal coordinates are important is in helping prove other things and so our goal is to come up with a proof that lands us with an equation for that explains $\boldsymbol{R}$. See what we're basically looking for is just like this we had an equation that tells us something about how to interpret the Ricci tensor. We're looking for a similar equation for $\boldsymbol{R}$ and to get there it's actually a pretty lengthy process but to get there we have to go through this concept of Riemann normal coordinates and basically what we're going to say is, let's imagine we have a situation in space-time let's erect a set of Riemann normal coordinates at that point so that's a very loaded phrase you need to understand what Riemann normal coordinates are and believe that they can be put and we can always put them and establish them in any space-time so that's going to be the focus of this lecture and then once we do that in our next lecture we'll start the long march to the curvature scalar, let's begin.

We'll begin by considering the space of Special relativity so this is all simply Special relativity, another way of thinking of that is simply that the metric of space-time $g_{\mu\nu}$ is is given by $\mathrm{diag}(-1,1,1,1)$, the diagonal elements of this tensor and that's true everywhere so there's no gravity, no curvature anywhere in this story so I'll align the time axis of our global coordinate system, we have some global coordinate system in our space-time, let's just say you could say it's just a Cartesian system and we have the X^1 coordinate system here and we'll suppress the other two dimensions for now and we're gonna talk about movement that's arbitrary in this space-time time-like but arbitrary so we'll assume of course that $c=1$ and so time-like movement would be constrained to this 45º light cone right, and we're measuring time in units of distance so you've got to be up pretty good with that, you have to have the idea in your head that time is measured in units of distance and that's because the fundamental constant of nature the speed of light $c=1$. In fact that's so important that I'm going to erase this and make it look much better.



When you do that and you draw graphs of it you end up with light cones that have 45º of angle which is nice because you can now determine immediately what is a future directed time-like path so now we take our particle here and we allow it to have an arbitrary movement as long as it's time-like right now, notice what I just drew that does not satisfy, I just drew a really bad time line curve, just because it's contained in the forward light cone doesn't mean it's time-like right obviously if you blew up this part of it, it's moving from this point in space to this point in space much faster than the speed of light right.

This curve is a disaster so I've already screwed up this explanation so let me undo that curve and the idea is we need to keep the curve so it's the path is less than 45º so for example, if it was just sitting still it'd be going like this right a straight line, sitting still in its own reference frame obviously but then it's allowed to move but it can never move beyond 45º in slope right so if I started doing that that'd be bad right so I can do this and this but it's so easy to screw this up right so let's just do this but ah there it is screwed up again. Regardless it can be otherwise arbitrary that's not even good right right so here let's say it turns on its engine and it's really kissing the speed of light there but it doesn't right and then it comes back motion. The point is that this can be an arbitrary movement of arbitrary curve in space-time we're not forcing it to be an inertial frame in Special relativity, now in Special relativity the notion of an inertial frame is just the same as in any other elementary physics, rectilinear motion with a constant velocity counts as an inertial frame in Special relativity. In General relativity, of course, an inertial frame is any frame in free-fall right, any free-fall whatsoever counts as an inertial frame but in Special relativity it's always straight-line motion constant velocity.

That's not what's going on here right, this guy is straight-line motion at constant velocity here in principle you know maybe if you blew it up real carefully you'd realize oh he's still moving a little bit but assume that's a straight line so he's an inertial frame right here, he or she, and then there's some acceleration here and perhaps if that line is straight perhaps he's there's an inertial frame here of going very fast almost at the speed of light definitely relativistic speeds right there then there's acceleration here and it slows down but all in all this is not an inertial frame for this entire period of time.

Global Reference Frame

Now consider this laboratory, this particle that takes its own reference frame with it right? Right now we're looking at a global reference frame right, and this is an important point that hangs with us in General relativity too, these coordinates map out the entire space-time and there don't attach to any observer right, there's not necessarily an observer who has this coordinate system naturally attached to them in other words it doesn't represent, I guess the way to think of it, is this X^0 time axis does not necessarily have to represent the proper time of any observer. In Special relativity it almost always does. I'm in fact I can't think of a situation in Special relativity where the time axis doesn't represent the proper time of somebody, there is this light cone coordinates that exist in Special relativity and I guess that would not reflect the proper time of any observer but in this case, all right, I guess fair enough right in this case there is an observer here whose coordinate system is in fact defined by this X^0 and X^1 very well, I guess I could draw that observer right here and that person is stationary with respect to the center of the coordinate system so this scientists proper time is given by X^0 . In General relativity, by the way, just that does not always happen in fact it almost never happens that the coordinate systems that we use to globally define things really represents the proper time of anybody in particular, I mean it does happen right, the Schwarzschild coordinates represents the proper time of a far away observer but any version of it the Eddington-Finklestein coordinates and all those other coordinates, all the ones we talked about early in the course there's no observer who's proper time is associated necessarily with the X^0 axis but in Special relativity is not a problem in this case, you know, we have this guy who's this lab frame out there but the point is the thing about relativity is that the lab frame of this guy is no different in principle than the lab frame of this guy, there's nothing preferred about this frame so we want to talk a little bit about what is the lab frame of this cat.

If you went into there and you blew it up you would get a laboratory so let's let's do that let's say we blew this up and here's the little laboratory that's moving along that purple world line right and there's the little box that represents the laboratory now the reason I'm drawing a little box is I want to represent a small laboratory because in General relativity you can only consider laboratories that are so small that the space time inside there can't detect the curvature of the global space time. In Special relativity that doesn't matter there's no curvature to detect but let's for the sake of just anticipating moving into the curved space-time let's consider a small local laboratory that's got this motion right, this is the motion that it's following. Inside that laboratory you can create a frame of reference, you can create a vector frame, you can say I'm moving in time in the e_0 direction and then I have three space directions, e_1 , e_2 and then e_3 which I have to suppress, here remember this is space-time, it's four dimensional so I can erect my little coordinate system and draw it to explain everything but I have to say that the e_0 direction is the time direction and e_1 , e_2 are the space directions inside the little lab so I guess maybe the better way to do this is to take my little lab and suppress its dimension also right so my little lab actually looks like a square instead of instead of a little cube.

What's up with these these basis vectors? Now we've got this guy who lives inside this laboratory right here he is and this is the coordinate system he sets up now his partner, I guess I'll do the Alice/Bob so this is Bob right and this is Alice. Alice when she looks at his coordinate system at this point in time when he's not moving when Bob is not moving relative to Alice, Alice believes that the time vector that Bob sees is just equal to her basis vector, her X^0 basis vector, I should do these:

$$e_0 = \partial_{X_0} \equiv \partial_0 \quad , \quad e_1 = \partial_1 \tag{3}$$

They're aligned, this two time-like vectors are aligned so they're all the same. Alice's reference system and Bob's reference system are exactly the same. Now at some point Bob starts to accelerate, that point is right here where this curve starts to bend and at that point he has his coordinate system is going to be different than hers and it's going to be different by a Lorentz transformation, because this is a here that Bob has achieved some velocity and when you achieve a velocity in Special relativity your reference frame or the reference frames compared to the lab always changes by a Lorentz transformation and so the way we may treat that is we would write that Lorentz Transformation.

Lorentz Transformation

In this circumstance now this guy's got some velocity right he's Bob has accelerated for a period of time and now there's some velocity so the relationship between Bob's new basis vector, I'll call that $e_{0'}$ is equal to some Lorentz transformation, I guess it would be:

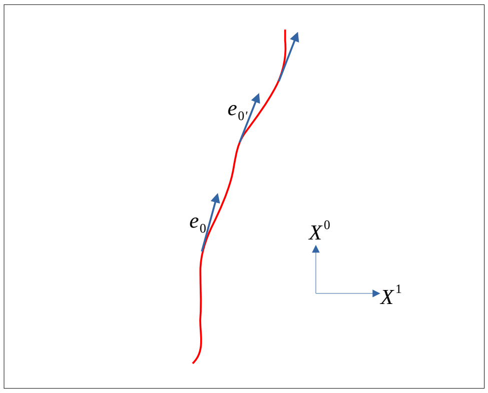
$$e_{0'} = \Lambda^{0}{}_{0'} \, e_0 \tag{4}$$

This is a Lorentz transformation that expresses that change. Now the problem with this is that there's actually a lot of Lorentz transformations that will represent this change because not only because Lorentz transformations don't just represent boosts right, Lorentz transformations represent boosts plus rotations right, so you can have many different Lorentz transformations that will change this time vector, this time-like basis vector, that will also induce some additional rotation right, it's not just going to be the velocity involved it will also be some arbitrary rotation and so you can now imagine, that not only would the time-like basis vector shifts and actually what it does is it tilts and rotates but there's some mixture between $e_{1'}$, $e_{1'}$ ends up equaling some, it starts to rotate too it rotates as well.

We don't want that, we want to just have the Lorentz transformation that all it does, the only thing that the Lorentz transformation does is it rotates e_0 so that e_0 is always pointing in the direction of the velocity, the 4-velocity right so when when I draw this velocity here I'm talking, it usually looks like oh it's moving, it's a velocity is in the X^1 direction and that is what it means in the Special relativity picture but the truth is that the velocity vector is actually off tangential to the world line right so if I blew up the velocity vector world line, the velocity vector here is in this direction but here it's in that direction and here, it's in this direction.

Velocity Vector

The velocity vector is always tangent I mean that's the definition of the velocity vector right it's tangent to the world line so what we want for our coordinate system is we want e_0 for the coordinate system of Bob has always got to be proportional to Bob's 4-velocity which would be $V^{\mu} \partial_{\mu}$ right? We want e_0 to always be proportional to the 4-velocity and that's another way of saying, that's just the fancy mathematical way of saying that this reference frame is not moving relative to Bob, Bob is fixed in this reference frame right, all of the velocity of this reference frame is wrapped up in the time vector and everything else it's still so this is the definition of a reference frame fixed relative to Bob. The time-like part is proportional to the 4-velocity so now when you look at that when you think of it in that those terms you say well so what gives?



Let me draw this a little bit cleaner so here's a slightly cleaner picture of this when Bob is motionless e_0 and the world line, well e_0 is always tangent to the world line so when he's motionless e_0 in the world line is vertical our picture has e_0 being vertical. There goes a period of brief acceleration and then the world line is now moving off, sloped off to the right not 45° because that would violate the speed of light but it's enough now that the velocity vector has now tilted in space-time right, remember this is all space-time so we have X^0 going this way and we have X^1 going that way.

Once it's tilted, we want his lab frame, we were interested in the frame that such that the basis vector is exactly aligned with the velocity vector and that's what I have here, this is to show you that this basis vector is now tangent to the velocity vector and what's the velocity vector well I'm sorry the basis vector is parallel to the velocity vector which is tangent to the world line so you're supposed to think of this as some tangent here, maybe if I draw a little dotted line here and a dotted line there and you see that that's tangent right? Obviously the Lorentz transformation we're interested in is a rotation that rotates e_0 to $e_{0'}$. Now everything's normalized right, because we do have a metric, this flat space-time as a metric right, we just usually call it $\eta_{\alpha\beta}$ right, it has a metric so everything is orthogonal and everything can be measured therefore everything can be normalized so we're assuming we have normalized basis vectors and we're interested in the Lorentz transformation that rotates e_0 to $e_{0'}$.

Now to think about a rotation of a vector in four-dimensional space-time is actually something that we have to rewrite our brains to get around because what we usually say is, I've got some body let's let me just draw some body in blue and this body whatever it is we're going to rotate it so I usually think of, if you're an engineer or as a physicist that's only studied the elementary Physics, you think of some vector and then you think of some angular velocity and you might call that vector or the angular velocity vector ω and ω points in a direction that's perpendicular to the plane of rotation so normally we think of the vector the angular velocity is perpendicular to the plane of rotation but we always fixate on this angular velocity vector and then we understand how something might rotate around that axis so we're thinking of rotation around an axis and it's also interpreted as a rotation around a plane of rotation and there's going to be some basis system, I guess it would be $\hat{i}, \hat{j}, \hat{k}$, those are the basis vectors and once you define ω in terms of $\hat{i}, \hat{j}, \hat{k}$, you decompose it in terms of the basis, you can find two vectors that define the plane of rotation which would be the plane completely orthogonal to ω.

Rotation in 3 dimensions

This is just really routine stuff that we get very used to when we try to understand rotations, the problem is that the relationship between a vector like this and a plane like this that can be defined unambiguously orthogonal to each other only exists in three dimensions, you cannot you cannot elevate that thinking into four dimensions or five dimensions or any other number of dimensions, I think there's a couple other examples of dimensions where you can do that but it's not relevant to any work and you might as well just presume that it can only be done in three dimensions and therefore when we think of a rotation of this vector here in four dimensional space, because that's what's happening we're taking this basis factor this the time-like basis vector presuming that the other three basis vectors are orthogonal to the time-like basis vectors ergo that means they are by definition space-like so the three other spaces basis factors here e_1, e_2, e_3 , they are definitely space-like because there are orthogonal to this one e_0 and this one's totally time-like by definition so the point is rotating this vector from this position to this position $e_{0'}$ in four-dimensional space, you can't think of a single axis that you can rotate around to do that, it doesn't exist the number of these vectors and the number of these planes is not the same except coincidentally in 3D. If you want a more deeper analysis of that, you go back to the what is a tensor series and in some of the later lectures I talk about this in depth where what's really going on here is this, if we call this a vector and these guys co-vectors, there's different spaces, different exterior power spaces that have different dimensionalities that only seem to equal in the case of 3D.

I don't want to go into it now but suffice to say you've got to get away from thinking about rotating around a vector however, you can always think about rotating in a plane right a plane of rotation is the fundamental way of thinking about rotations. If you can define a plane in any dimension, you can always think about rotating in that plane so in this case we need to be able to think of a plane of rotation what's the plane that e_0 is going to be rotating in to move from this orientation to this orientation $e_{0'}$ and when I talk about these two different orientations, I'm talking about it relative to this lab frame right should I always erase when I don't mean to what, I need to point I'm talking about the rotation relative to the lab frame because all of these vectors for right now are going to be defined in terms of the lab frame so e_0 is going to equal yeah e_0 which is the time-like basis vector of Bob is going to be defined analysis universal frame which is allowable in Special relativity to be something like, let's see something like what would it would be:

$$e_0 = \alpha^0 \partial_0 + \alpha^1 \partial_1 + \alpha^2 \partial_2 + \alpha^3 \partial_3 \tag{5}$$

We know that in the case where Bob is stationary in Alice's frame which is exactly this area right here, this just equals ∂_0 , all the coefficients are 0 or 1 and then $e_{0'}$ is just some other set of numbers:

$$e_{0'} = \beta^0 \partial_0 + \beta^1 \partial_1 + \beta^2 \partial_2 + \beta^3 \partial_3 = \beta^{\mu}_{0'} \partial_{\mu} \tag{6}$$

Basis vectors

Notice that I could write that as write that this way $\beta^{\mu}_{0'} \partial_{\mu}$ where this expression $\beta^{\mu}_{0'}$ where $0'$ is not an index in this expression, the index is μ , this $0'$ is just naming the vector, this is the name of a vector e_0 is the name of the basis vector that points in the time-like direction or the 4-velocity direction of Bob and e_0 , there's four of those different vectors $e^0_0, e^1_0, e^2_0, e^3_0$ and they are themselves vectors that have to be expressed in terms of a basis and the basis we're using is Alice's basis in this case so when I write $e_{0'}$ that's another vector, now it happens to be Bob's basis vectors so in his world he'll end up calling that something like ∂^{B}_0 , this will be ∂_0 in Bob's frame, in principle if he was to do experiments based on it. Anyway the point is that we need to understand the plane of rotation that defines the movement of this time-like basis vector for Bob and that plane of rotation well, it certainly is a plane that contains e_0 , we don't want to change the length or size of e_0 so whatever that plane is, it's actually going to contain e_0

which we would think about in terms of, if we had a plane and there was a vector in the plane and we want to rotate that vector in the plane, we want the plane that and we don't want the vector to change size, the plane will have to contain the vector so to define this plane we were halfway there, we need two vectors to define a plane but we've got one, the point is is what's the other vector that's lying in the same plane that will unambiguously define this plane as opposed to say there's the same vector or say this plane right which might be a little more vertical through it or something like that right.

What's the second vector that is unambiguously defined by this world line I'll let you think about this for a second maybe you want to pause the video and think about it this world line is clearly defined by its tangent vector right but that doesn't unambiguously define a plane right it just defines a single vector there's an infinite number of planes that contain e_0 and I could rotate through all of them but I only want to rotate through one of them and that one will give us this new vector here what is the other vector that's unambiguously defined by this world line that I can use to define the plane of rotation and the answer is it's the acceleration vector right it's the acceleration vector, the 4-acceleration a^{μ} and the 4-acceleration is always unambiguously defined by the rule that the 4-velocity dotted with the 4-acceleration, always equals zero right:

$$\vec{V}\cdot\vec{a}=0 \tag{7}$$

In Special relativity, this is such a critical point to understand, the 4-velocity of anything at any time in any place dotted into the 4-acceleration of that object is always going to be zero and the reason this is the case is that the magnitude of the 4-velocity of anything any time at any place is always equal to the speed of light which in our situation is 1. The magnitude of any object, any energy anything in the entire universe, the magnitude of its 4-velocity is always equal to 1 and yet things do accelerate right, they do accelerate but they accelerate in such a way that the magnitude of the 4-velocity never changes ergo all 4-accelerations must obey this principle right here that the 4-acceleration dotted into the 4-velocity is equal to zero, that means the 4-acceleration can never be and it can never have any component in the direction of the 4-velocity and of course that makes sense so imagine you're in Bob's frame right, you're sitting in Bob's frame and you're proceeding along in space-time with a 4-velocity that is you erect a frame where the e_0 is in your time direction so it's basically ticking off units of a clock right in units of meters in our example right if you're for acceleration had a component along that direction then all of a sudden you would find yourself moving quicker through time you could build a
time machine you could move quicker or slower through time you could accelerate and turn your time vector into the
opposite direction right and we can't do that right we cannot change our rate in
our laboratory of going through time right obviously the Lorentz transformation makes clocks tick at different rates but if this were possible your own experience of time
would suddenly start to change and that's crazy well it's also just not the
way things are right Special relativity ultimately this is a consequence of the
fundamental assumption that there's no preferred reference frame that the speed of light is measured to be the same by
everybody you know regardless of their state of motion all of that leads to this conclusion we interpret this
conclusion is to say you can never accelerate your way through time so we
For acceleration
can look we can look at this situation here right where there's our
4-velocity well what's our for acceleration well it's got to be orthogonal to it so we're assuming that

we're only accelerating the X direction right the y&z direction have been isolated from the problem so there
there's B for acceleration so that's a and that's easy Rho and that defines the
plane of rotation which in our picture is very satisfactory right it that's exactly the plane for which easier does
in fact rotate and then at this point the for acceleration always has to be orthogonal so it's always still going to
have to be perpendicular to this so if
we so the belt out of this is if we find the Lorentz transformation that moves
that only rotates in that plane and that's the only Lorentz transformation
we execute that is so we advance down
the world line and then we re-establish a reference frame for Bob based entirely
on the Lorentz transformation that is a rotation in the easy row a plane right
so it's got to be a little infinitesimal advance down the world line subsequent when we we do the advance down the world
line a little bit and then we rotate easy row in the easy row alpha
acceleration plane that is called Fermi Walker transport of of this reference frame that
reference franklin Fermi walker transported and understand what this is this is this Fermi walker transport is
Fermi Walker Transport
relevant to Alice right who's out here watching this Alice has to decide what
does the reference frame of Bob look like you know Bob is sitting here in his reference frame after Bob has moved an
infinitesimal amount down his world line right i know what his basis vectors were
here says Alice because they were the same as mine cuz he was not moving relative to me but then he starts to
accelerate well now I have two Lorentz transform his basis system well I'm
gonna do that in the most efficient way possible I'm going to do the Lorentz transform that rotates his time-like
basis factor in what I understood to be his his time light basis vector the
plane that was or that was defined by the time light basis vector and the acceleration that I've observed and then
I recalculate and now I know what his new basis system looks like and I can be
assured because I forced the rotation to be in this plane that there was no unnecessary rotation of just the spatial
basis vectors those guys do get rotated but they don't get rotated beyond any
more than is absolutely necessary to realign his Bob's spatial or his Bob's
time-like basis vector with his purely with his 4-velocity right he realigns
his timeline factor with his 4-velocity so that is Fermi Walker
transport and that exists entirely in the Special relativity world but notice
Special Relativity
what's nice about it is you know Special relativity we usually you know do all our paradoxes and our introductions and
things like that our basic study of it comparing two different inertial frames
but this allows us to talk about an arbitrarily accelerated observer right we can figure out what that observers
laboratory frame looks like regardless there's their state of motion what we're
doing those we're saying there's only infinitesimal changes this from here to here in order to do a Lorentz transformation from here to here I've

got to consider just this instant of motion right this instant of motion and if this acceleration is very very
complex I've got to make sure that the the step by step in form infinite tests
I've got to do this in infinitesimal steps right I've got to do this along the world line but very very carefully
to make sure that the four acceleration is constant along the the period of time
that I am engaging in this this this flip or this this transformation so
that's Fermi walk or transport so I think I'll end it there because to now
Curved Spacetime
we have to take Fermi Walker transport and understand it in terms of curved space-time and that's what Riemann
normal coordinates are Ramon normal coordinates are understanding so Fermi Walker transport in the context of
curved space-time and the complications that are introduced by curved space-time is well first of all you don't have the
coordinate system you're dealing with isn't necessarily the proper time of Alice so Alice has to be removed
from the picture as a separate observer so that's one little issue that's not a tough one and then on top
of that we have to understand that the lab has to be very small relative to
relative to all of the geodesics and separation of space-time in other words
you can't detect gd's exceed separation in a lab because it's so tiny and and
the last problem seems to be it's it's a the last problem is a bit of a
challenge to even understand because what we when we look at this situation
we understand the direction x1 and x0 we understand the orthogonality we really
understand how to define a reference frame for everybody involved but it's
much more complex when you have an arbitrary coordinate system in a curved space-time you have to say well what does it mean like like
here you let's say Bob looks out in the x1 direction we know where he's gonna see forever never never in curved
space-time this there's no straight line and yet well there is a straight line in the x1 in direction but
it's a geodesic line so we have to understand how that notion affects everything and it does it's a heavy
heavy heavily affects everything but it is still the and also the idea of being
an inertial observer is much more complicated because in curved space-time because in Special relativity we just
need constant motion along a straight line in General relativity we're basically talking about an observer in
free-fall and so we have to establish free-fall coordinates in that are match
along the world line infinitesimally you see what we've done here is we've said at this moment there's some reference
frame that's got the exact same floor velocity is Bob and that reference frame at that instant has this coordinate
system and that reference frame at that instant is inertial but the next instant it's not Bob has moved on into a new
reference frame because Bob's motion is not inertial Bob's got this curved motion here and so another reference
frame exists that's instantaneously Co moving with Bob but is inertial and
understanding that in curved spacetimes just a little little trickier so we'll talk about that next time all right
see